

The University of Texas at Austin  
Dept. of Electrical and Computer Engineering  
*Final Exam*

Date: December 17, 2005

Course: EE 313 Evans

Name: \_\_\_\_\_  
Last, First

- The exam is scheduled to last 3 hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

| Problem | Point Value | Your score | Topic                        |
|---------|-------------|------------|------------------------------|
| 1       | 10          |            | Differential Equation Rhythm |
| 2       | 10          |            | Differential Equation Blues  |
| 3       | 10          |            | Stability in Two Domains     |
| 4       | 10          |            | Convolution in the Abstract  |
| 5       | 10          |            | Sampling in Continuous-Time  |
| 6       | 15          |            | Digital Filter Analysis      |
| 7       | 15          |            | Digital Filter Design        |
| 8       | 10          |            | Amplitude Modulation         |
| 9       | 10          |            | Amplitude Demodulation       |
| Total   | 100         |            |                              |

**Final Exam Problem 1.** Differential Equation Rhythm. 10 points.

For a continuous-time system with input  $x(t)$  and output  $y(t)$  governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 12y(t) = x(t)$$

for  $t \geq 0^+$ .

- (a) What are the characteristic roots of the differential equation? 2 points.
- (b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of  $C_1$  and  $C_2$ . 4 points.
- (c) Find the zero-input response for the initial conditions  $y(0^+) = 0$  and  $y'(0^+) = 1$ . 4 points.

**Final Exam Problem 2.** Differential Equation Blues. 10 points.

For a continuous-time linear time-invariant (LTI) system with input  $x(t)$  and output  $y(t)$  governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 12y(t) = x(t)$$

for  $t \geq 0$ .

(a) What is the transfer function? 2 points.

(b) What are the values of the poles and zeroes of the transfer function? 2 points.

(c) What is the region of convergence for the transfer function? 2 points.

(d) Give a formula for the step response of the system. 4 points.

**Final Exam Problem 3.** Stability in Two Domains. 10 points.

In this problem, the input signal is denoted by  $x(t)$  and the output signal is denoted by the output signal  $y(t)$ .

(a) Is the system defined by  $\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 12y(t) = x(t)$  asymptotically stable, marginally stable, or unstable? Why? 3 points.

(b) Convert the differential equation in (a) into a difference equation by first substituting the approximations  $\frac{d}{dt} y(t) \approx \frac{y(t) - y(t - T_s)}{T_s}$  and  $\frac{d^2}{dt^2} y(t) \approx \frac{y(t) - 2y(t - T_s) + y(t - 2T_s)}{T_s^2}$  and then sampling at  $t = n T_s$ , where  $T_s$  is the sampling period. 3 points.

(c) Is the difference equation you derived in part (b) asymptotically stable, marginally stable, or unstable? Why? 4 points.

**Final Exam Problem 4.** Convolution in the Abstract. 10 points.

(a) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false:* The convolution of a finite duration discrete-time signal (other than a signal that is identically zero for all time) and an infinite duration discrete-time signal always produces an infinite duration discrete-time result. 5 points.

(b) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false:* The continuous-time convolution of two signals  $f(t)$  and  $g(t)$  can always be computed by taking the inverse Laplace transform of  $F(s)G(s)$ . You can assume that  $F(s)$  and  $G(s)$  exist. 5 points.

**Final Exam Problem 5.** Sampling in Continuous Time. 10 points.

Sampling of an analog continuous-time signal  $f(t)$  can be modeled in continuous-time as

$$y(t) = f(t) p(t)$$

where  $p(t)$  is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

such that  $T_s$  is the sampling duration. The Fourier series expansion of the impulse train is

$$p(t) = \frac{1}{T_s} (1 + 2 \cos(\omega_s t) + 2 \cos(2 \omega_s t) + \dots)$$

where  $\omega_s = 2 \pi / T_s$ .

(a) Plot the impulse train  $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ . 2 points.

(b) Find  $P(\omega)$ , the Fourier transform of  $p(t)$ . 2 points.

(c) Express your answer for  $P(\omega)$  in part (b) as an impulse train in the Fourier domain. 3 points.

(d) What is the spacing of the impulse train  $P(\omega)$  with respect to  $\omega$ ? 3 points.

**Final Exam Problem 6.** *Digital Filter Analysis.* 15 points.

A causal discrete-time linear time-invariant filter with input  $x[n]$  and output  $y[n]$  is governed by the following difference equation:

$$y[n] = 0.8 y[n-1] + x[n] - 1.25 x[n-1]$$

(a) Draw the block diagram for this filter. 3 points.

(b) What are the initial conditions? What values should they be assigned? 3 points.

(c) Find the equation for the transfer function in the  $z$ -domain including the region of convergence. 3 points.

(d) Find the equation for the frequency response of the filter. 3 points.

(e) Is this filter lowpass, bandpass, bandstop, highpass, notch, or allpass? Why? 3 points.

**Final Exam Problem 7. Digital Filter Design.** 15 points.

Digital Subscriber Line (DSL) systems transmit voice and data over a telephone line using frequencies from 0 Hz to 1.1 MHz. DSL systems use a sampling rate of 2.2 MHz.

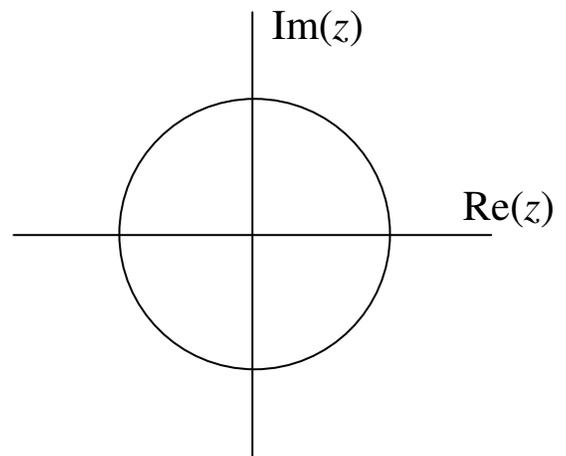
Consider an AM radio station that has a carrier frequency of 550 kHz, has a transmission bandwidth of 10 kHz, and is interfering with DSL transmission.

Design a digital filter *biquad* for the DSL receiver to reject the AM radio station but pass as much of the DSL transmission band as possible. A biquad has two poles and 0, 1, or 2 zeros.

(a) Is the digital IIR filter biquad lowpass, bandpass, bandstop, highpass, notch, or allpass? Why? 3 points.

(b) Give formulas for the locations of the poles and zeros of the biquad. 5 points.

(c) Draw the poles and zeros on the pole-zero diagram on the right. 4 points.



(d) Compute the scaling constant (gain) for the filter's transfer function. 3 points.

**Final Exam Problem 8.** Amplitude Modulation. 10 points.

In practice, we cannot generate a two-sided sinusoid, but we can generate a one-sided sinusoid.

Consider a one-sided cosine  $c(t) = \cos(2\pi f_c t) u(t)$  where  $f_c$  is the carrier frequency (in Hz).

(a) By using the Fourier transforms of  $\cos(2\pi f_c t)$  and  $u(t)$  from a lookup table, compute the Fourier transform of  $c(t) = \cos(2\pi f_c t) u(t)$  using Fourier transform properties. 3 points.

(b) Draw  $|C(\omega)|$ , the magnitude of the Fourier transform of  $c(t)$ . 3 points.

(c) Describe the differences between the magnitude of the Fourier transforms of a one-sided cosine and a two-sided cosine. What is the bandwidth of each signal? 4 points.

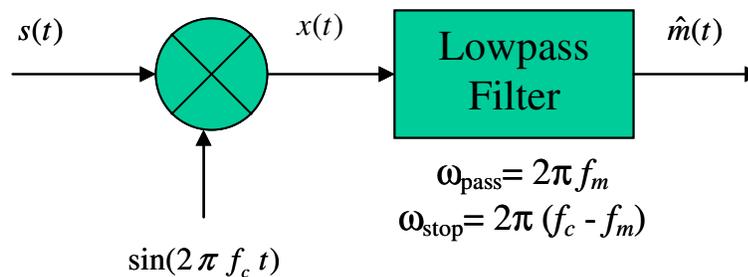
**Final Exam Problem 9.** Amplitude Demodulation. 10 points.

This problem explores the situation in an amplitude modulation system in which the receiver is 90 degrees out of phase with the transmitter.

A lowpass, real-valued message signal  $m(t)$  with bandwidth  $f_m$  (in Hz) is to be transmitted using amplitude modulation

$$s(t) = m(t) \cos(2 \pi f_c t)$$

where  $f_c$  is the carrier frequency (in Hz) and  $f_c \gg f_m$ . The receiver has metaphysical knowledge of the carrier frequency, but is 90 degrees out of phase with the transmitter. The receiver processing of the transmitted signal  $s(t)$  to obtain an estimate of the message signal,  $\hat{m}(t)$ , follows:



Hence,  $x(t) = s(t) \sin(2 \pi f_c t)$ .

What is the value of  $\hat{m}(t)$  ?